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SELECTION OF A WING SHAPE FOR HYPERSONIC VELOCITIES

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ABSTRACT: The selection of the shape and general dimensions of a wing moving at hypersonic speed is considered as an extremal problem with respect to minimum drag at a given lift, volume, leading edge temperature, and other parameters. The characteristic parameters on which the optimum profile of the longitudinal cross-section depend are determined with emphasis on their values, which separate those cases of known and unknown volume. It is shown that there exists a relative wing thickness for which the lift-drag ratio attains the maximum value at a given detrimental drag, and also that a spherical segment is not always the optimum shape for a leading edge in the stagnation point region. The Newton "law" for pressure and the "average" friction coefficient, which is independent of the wing shape, are used for solving the problem. Certain general characteristics of the wing which depend on the angle of attack are considered. Estimates of maximum possible values of lift-drag ratio obtained here are very close to those obtained by D. Kuchemann, thus permitting a verification of the results obtained according to Newton's law.

The selection of wing shape and dimensions is considered as an extremal /40[1] problem with respect to minimum drag at a given lift, volume, leading edge temperature and other parameters. The characteristic parameters which determine the optimum profile of the longitudinal cross-section are determined, and in particular their values which separate cases of known and unknown volume.

It is shown that there is a relative wing thickness for which the lift-drag ratio assumes a maximum value when the detrimental drag is assigned. A spherical segment is not always the optimum shape for the leading edge in the vicinity of the stagnation point.

The selection of wing shape and dimensions can be formulated as an extremal problem for drag, lift-drag ratio, maximum surface temperature or coolant consumption when the volume, some of the above quantities or other relations are specified.

It becomes necessary to use the simplest "laws" to solve such an extremal problem in a sufficiently general form without systematic large-volume computations. The present article adopts Newton's Law for pressure and the "average"

[1] Numbers in the margin indicate pagination in the foreign text.

coefficient of friction which does not depend on the shape of the wing. Naturally, in this case, shapes with surface breaks are not considered and the effects produced by the interaction of elements, shock waves, and the boundary layer also cannot be taken into account. We should bear in mind that in hypothetical flying machines it is difficult to separate the wing from the engine. The above statements define the solutions of extremal problems as limiting estimates of characteristics and general indicators rather than direct practical recommendations.

If the wing volume is not known we can assume that its lower part is plane, and that the upper part is in the "shadow" (the region of break-away flow). Then according to Newton's Law the lift, drag, and lift-drag ratio are given by the following

$$Y = 2qS \sin^2 \alpha \cos \alpha, \qquad X = 2qS(c_0 + \sin^3 \alpha)$$
$$K = \sin^2 \alpha \cos \alpha \,/\, (c_0 + \sin^3 \alpha)$$

It is assumed that the coefficient of "detrimental" drag c_0 includes resistance to friction and the drag of the blunt edge of the wing as well as of its upper side and that this coefficient does not depend on the angle of attack. The maximum lift-drag ratio is given by

$$K_m = {}^1/_3(2 \operatorname{ctg} \alpha_m - \operatorname{tg} \alpha_m)$$

The corresponding angle of attack is determined from the relation

$$\sin \alpha_m \,/\, (2 \operatorname{ctg}^2 \alpha_m - 1) = c_0$$

The lift-drag ratio curves have a maximum value only if $\alpha_m < 54°44'$. This angle corresponds to the maximum possible lift and an infinite quantity c_0. The lift-drag ratio curves for all c_0 where $\alpha > \alpha_m$ are found between the curve ctgα, which gives the lift-drag ratio for an "ideal" plate (Fig. 1), and the curve K_m. It is obvious that a lift-drag ratio greater than 2 can be obtained only when the angles of attack are $\alpha < 20$-$25°$, i.e., when they are relatively small. This study pertains to a thin wing at low angles of attack.

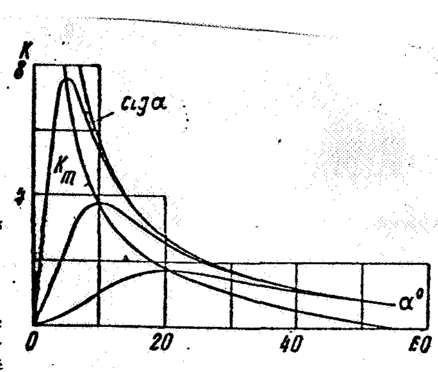

Figure 1

1. Profile of the longitudinal cross-section. The drag of the leading blunt edge is included at this time in the detrimental drag characterized by the coefficient c_0. We assume that the wing surface does not contain regions of finite area with zero pressure and we designate the lower and upper sides of the wing by the positive functions z_1 (x, y), z_2 (x, y). If we assume that the wing is thin (Fig. 2) we have /41

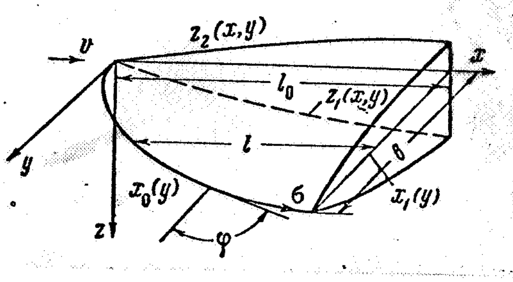

Figure 2

Then by disregarding infinitesimals of order greater than three we obtain the following expressions for lift and drag:

$$Y = 2q\int_0^b\int_{x_0}^{x_1}(z_{1x}^2 - z_{2x}^2)\,dx\,dy$$
$$\frac{X}{2q} = c_0 S + \int_0^b\int_{x_0}^{x_1}(z_{1x}^3 + z_{2x}^3)\,dx\,dy \qquad (1.1)$$

In the above equations S is the wing area and q is the impact pressure.

We shall seek the minimum value for drag when the lift and volume V are known, i.e., the minimum value of the functional[1]

$$X - \lambda Y - \frac{2q\mu V}{b} = 2q\Big[c_0 S + \int_0^b\int_{x_0}^{x_1}\Big(z_{1x}^3 - \lambda z_{1x}^2 - \frac{\mu z_2}{b}\Big)dx\,dy + $$
$$+ \int_0^b\int_{x_0}^{x_1}\Big(z_{2x}^3 + \lambda z_{2x}^2 - \frac{\mu z_2}{b}\Big)dx\,dy\Big] \qquad (1.2)$$

When the first variation of (1.2) is equal to zero we have

$$\frac{\partial}{\partial x}(3z_{1x}^2 - 2\lambda z_{1x}) + \frac{\mu}{b} = 0, \quad \frac{\partial}{\partial x}(3z_{2x}^2 + 2\lambda z_{2x}) + \frac{\mu}{b} = 0 \qquad (1.3)$$

The boundary values at the edges have the form

$$[(3z_{1x}^2 - 2\lambda z_{1x})\delta z_1]_{x_0}^{x_1} = 0, \qquad [(3z_{2x}^2 + 2\lambda z_{2x})\delta z_2]_{x_0}^{x_1} = 0 \qquad (1.4)$$

If we assume that the coordinates z_1, z_2 at the leading edge are known and that we have natural boundary conditions at the trailing edge, we obtain the following for the trailing edge:

$$z_{1x}(3z_{1x} - 2\lambda) = 0, \qquad z_{2x}(3z_{2x} + 2\lambda) = 0 \qquad (1.5)$$

Integrating (1.3) and determining the constants from (1.5) we obtain

$$z_{1x} = {}^1/_3\lambda + \sqrt{{}^1/_9\lambda^2 + {}^1/_3\mu b^{-1}(x_1 - x)},$$
$$z_{2x} = -{}^1/_3\lambda + \sqrt{{}^1/_9\lambda^2 + {}^1/_3\mu b^{-1}(x_1 - x)} \qquad (1.6)$$

[1] The problem of the wing of minimum drag is considered in [1,2].

Of the four roots of (1.5) only two satisfy the conditions of the problem

$$z_{1x} = {}^2/_3\,\lambda, \qquad z_{2x} = 0$$

i.e., the pressure becomes equal to zero at the trailing edge of the upper part of the wing. When the roots are selected in this manner the plus sign should be retained in front of the radicals.

Integrating (1.6) and assuming that the coordinates z_{10}, z_{20} at the leading edge are known, we obtain /42

$$\begin{aligned} z_1 - z_{10} &= {}^1/_3\lambda\,(x - x_0) + 2(b/\mu)\,\{[{}^1/_9\lambda^2 + {}^1/_3\mu b^{-1}(x_1 - x_0)]^{3/2} - \\ &\quad - [{}^1/_9\lambda^2 + {}^1/_3\mu b^{-1}(x_1 - x)]^{3/2}\} \\ z_2 - z_{20} &= -{}^1/_3\lambda\,(x - x_0) + 2(b/\mu)\,\{[{}^1/_9\lambda^2 + {}^1/_3\mu b^{-1}(x_1 - x_0)]^{3/2} - \\ &\quad - [{}^1/_9\lambda^2 + {}^1/_3\mu b^{-1}(x_1 - x)]^{3/2}\} \end{aligned} \tag{1.7}$$

We note that the factor $\mu < 0$ does not satisfy the conditions of the problem because in this case $z_{2x} < 0$. The extremal profile is symmetric and the angle of attack is $\alpha = 1/3\lambda$. Let us now compute X, Y, V:

$$\frac{Y}{2qb^2} = c_y = \lambda^2 f_2(\omega), \quad \frac{X}{2qb^2} - \frac{c_0 S}{b^2} = \lambda^3 f_3(\omega), \quad \frac{V}{b^3} = v = \lambda f_1(\omega) \tag{1.8}$$

Here

$$\begin{gathered} f_1(\omega) = ({}^2/_3)^3 \frac{1}{15\omega^2}\left(1 + {}^3/_2 i_{5/2} - {}^5/_2 i_{3/2}\right), \quad f_2(\omega) = ({}^2/_3)^3 \frac{1}{3\omega}\left(i_{3/2} - 1\right) \\ f_3(\omega) = ({}^2/_3)^3 \frac{1}{5\omega}\left({}^1/_6 i_{5/2} + {}^5/_6 i_{3/2} - 1\right) \\ i_n(\omega) = \int_0^1 (1 + 3\omega l')^n\, dy', \qquad \omega = \frac{\mu}{\lambda^2}, \qquad l' = \frac{l}{b} = \frac{x_1 - x_0}{b} \end{gathered} \tag{1.9}$$

The functional (1.2) is equal to

$$X - \lambda Y - \frac{2q\mu V}{b} = 2qb^2\left[c_0 \frac{S}{b^2} + \frac{({}^2/_3)^3\lambda^3}{15\omega}\left(1 - i_{5/2}\right)\right] \tag{1.10}$$

The ordinate of the profile for the longitudinal cross section of the wing is

$$z' = \frac{z - z_0}{b} = \frac{2\lambda}{27\omega}\left\{(1 + 3\omega l')^{3/2} - [1 + 3\omega l'(1 - \xi)]^{3/2}\right\}, \quad \xi = \frac{x - x_0}{l} \tag{1.11}$$

It is obvious from equations (1.8) that, under the assumed conditions, the characteristics of the wing are affected only by the distribution of wing area across the span and not by the plan form. The value of parameter ω, which determines the shape of the profile, depends only on the parameter

(1.12)

which is characteristic for variational problems in which the minimum is sought when the lift and volume are known[1].

The problem of the maximum lift-drag ratio for a known volume, i.e., the problem of the minimum value of the functional $X/Y-\nu V/b^3$, is reduced to the preceding one if we assume that $\lambda = 1/K$, $\mu = \nu Y/2qb^2$, where ν is an indeterminate factor.

Equations (1.8) acquire the form

$$\nu K = f_1(\omega), \quad \nu = \frac{\omega}{f_2(\omega)}, \quad \frac{\omega}{\nu} - \frac{K^3 c_0 S}{b^2} = f_3(\omega)$$
$$\frac{c_0 S}{b^2 \nu^3} = \frac{f_2(\omega) - f_3(\omega)}{f_1^3(\omega)}, \quad \omega' = \nu c_y K^2 \tag{1.13}$$

Thus, in the given case, the characteristic parameter which determines the shape of the profile will be the ratio of the "detrimental" drag to the wave drag[2] $c_0 S/b^2 \nu^3$. The functional is /43

$$X/Y - \nu V/b^3 = [1 - \nu/f_1(\omega)]/K \tag{1.14}$$

When $\omega \to 0$ expression (1.11) for z' becomes

$$z' = {}^1\!/_3 \lambda (x - x_0) / b$$

i.e., the profile changes to a wedge with the upper side along the flow.

We use the expansion $(1 + 3\,\omega l')^n$ to find the values of the characteristic parameters corresponding to ω = 0.

By integrating we obtain

$$i_n(\omega) = 1 + n(3\omega)s_1 + {}^1\!/_2 n(n-1)(3\omega)^2 s_2 + {}^1\!/_2 \cdot {}^1\!/_3 n(n-1) \times$$
$$\times (n-2)(3\omega)^3 s_3 + {}^1\!/_2 \cdot {}^1\!/_3 \cdot {}^1\!/_4 n(n-1)(n-2)(n-3)(3\omega)^4 s_4 + \ldots$$
$$s_n = \int_0^1 l'^n\, dy', \quad y' = \frac{y}{b}$$

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[1] This parameter is discussed in [7] which was generously provided by the author.

[2] This parameter is characteristic of all variation problems in which the volume and "detrimental" drag are known [1,4].

Substituting this expression into (1.9) we have

$$f_1(\omega) = {}^1/_3(s_2 + \omega s_3 + \ldots), \quad f_2(\omega) = {}^1/_3({}^4/_3 s_1 + \omega s_2 + \ldots)$$
$$f_3(\omega) = {}^1/_3({}^8/_9 s_1 + \omega s_2 + \ldots)$$

and from (1.2) we obtain

$$\frac{v^2}{c_y} = \left(\frac{s_2^2}{4s_1}\right)\left[1 + \omega\left(\frac{2s_3}{s_2} - \frac{3s_2}{4}\right) + \ldots\right] \qquad (1.15)$$

Consequently $\omega = 0$ when $v^2/c_y = s_2^2/4s_1$. Similarly, from (1.13) we have

$$b^2v^3/c_0S = (s_2^3/4s_1)(1 + 3s_3\omega/s_2 + \ldots)$$

and ω becomes equal to zero when

$$b^2v^3/c_0S = s_2^3/4s_1 \qquad (1.16)$$

The profile of the longitudinal cross-section is symmetric and convex (optimum distribution of volume on both sides of the velocity vector) when

$$4s_1v^2/s_2^2c_y > 1, \quad 4v^3/c_0s_2^3 > 1$$

In order to complete the computations it is necessary to use the function $l'(y')$. We will apply it in the form $l' = l_0'(1 - y')^r$. Integrals i_n can be computed in finite form when r = ... 1/2, 2/3, 1, 2. Let us compute them for r = 1 and r = 1/2,

In the case when r = 1

$$\omega' = \omega l'$$

$$f_1'(\omega') = \frac{f_1(\omega')}{l_0'^2} = \frac{({}^2/_3)^3}{15\omega'^2}\left[1 + \frac{(1+3\omega')^{7/2} - 1}{7\omega'} - \frac{(1+3\omega')^{5/2} - 1}{3\omega'}\right]$$

$$f_2'(\omega') = \frac{f_2(\omega')}{l_0'} = \frac{({}^2/_3)^3}{3\omega'}\left\{\frac{2}{15\omega'}[(1+3\omega')^{5/2} - 1] - 1\right\}$$

$$f_3'(\omega') = \frac{f_3(\omega')}{l_0'} = \frac{({}^2/_3)^3}{5\omega'}\left[\frac{(1+3\omega')^{5/2} - 1}{9\omega'} + \frac{(1+3\omega')^{7/2} - 1}{63\omega'} - 1\right]$$

In the case when r = 1/2

$$f_1'(\omega') = \frac{({}^2/_3)^3}{15\omega'^2}\left\{1 + \frac{4}{9\omega'^2}\left[\frac{(1+3\omega')^{9/2} - 1}{6} + \frac{(1+3\omega')^{5/2} - 1}{2} - \frac{(1+3\omega')^{7/2} - 1}{{}^7/_4}\right]\right\}$$

$$f_2'(\omega') = \frac{({}^2/_3)^3}{3\omega'}\left\{\frac{4}{9\omega'^2}\left[\frac{(1+3\omega')^{7/2} - 1}{7} - \frac{(1+3\omega')^{5/2} - 1}{5}\right] - 1\right\}$$

$$f_3'(\omega') = \frac{({}^2/_3)^3}{5\omega'}\left\{\frac{4}{27\omega'^2}\left[\frac{(1+3\omega')^{7/2} - 1}{{}^7/_2} - \frac{(1+3\omega')^{5/2} - 1}{2} + \frac{(1+3\omega')^{9/2} - 1}{18}\right] - 1\right\}$$

The limiting values of the characteristic parameters are /44

$$g_1 = \frac{v^2}{c_y l_0'^3} = \begin{cases} 1/18 & (r = 1), \\ 3/32 & (r = 1/2), \end{cases} \qquad g_2 = \frac{c_0 S l_0'^5}{b^2 v^3} = \begin{cases} 54 & (r = 1) \\ 64/3 & (r = 1/2) \end{cases} \tag{1.17}$$

Figure 3 shows the variation in ω', f_1', f_2', f_3' as a function of the characteristics parameter g_1; as this parameter increases the magnitude of ω' increases without bounds as the square of this parameter, and when this parameter is large the wing profile of minimum drag, where the volume of the wing is known [1,2], is

$$z' \sim 1 - (1 - \xi)^{3/2}$$

When the wing has a large area in plan form ($r = 1/20$), with the same characteristic parameter, the profile is less convex and changes into a wedge when the values of the characteristic parameter are large. Figure 4 shows the variation in ω' and f_1' as a function of the characteristic parameter g_2. The interesting difference from the preceding case is that the parameter varies within defined bounds.

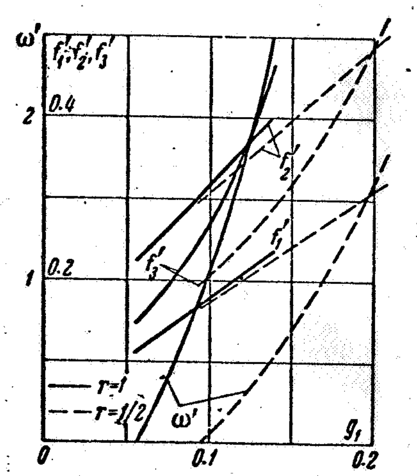

Figure 3.

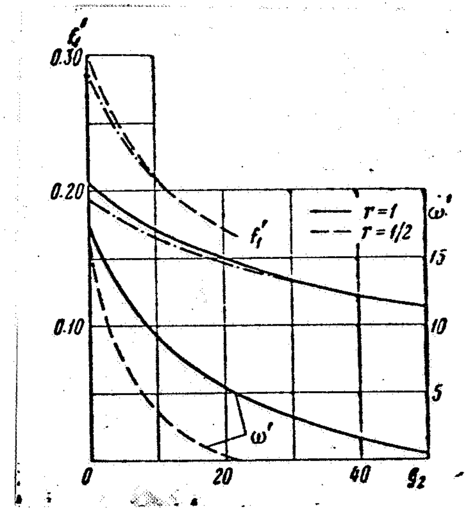

Figure 4.

Thus the optimum profile of the longitudinal cross-section of the wing remains convex when the characteristic parameter v^2/c_y varies from ∞ to $s_2^2/4s_1$ and the parameter c_0S/b^2v^3 varies from zero to $4s_1s_2^3$. When the characteristic parameters have values of $s_2^2/4s_1$, and $4s_1/s_2^3$, the optimum profile becomes a wedge with the upper side directed along the flow. As the characteristic parameters are varied further the rectilinear lower generatrix remains optimum while the upper one in the "shadow" becomes arbitrary. It should be noted that under the assumptions of a thin wing, a displacement of the longitudinal cross-section in plane yz, where their slopes remain constant, has no effect on the characteristics of the wing. Consequently the shape of transverse cross-sections may be selected in such a way as to satisfy some other requirement, for example, concerning the distribution of thermal flux along the transverse cross-section.

2. Area, width and span of a wing. Let us consider several general variations in the wing characteristics as a function of the angle of attack. For an arbitrary profile of longitudinal cross-section we have

$$Y = 2q\left[i_{12} - i_{22} + 2\alpha(i_{11} + i_{21})\right]$$

$$X = 2q\left[c_0S + i_{13} + i_{23} + 3\alpha(i_{12} - i_{22}) + 3\alpha^2(i_{11} + i_{21})\right]$$

$$\left(i_{n\nu} = \int_0^b \int_{x_0}^{x_1} z_{nx}{}^{\nu}\, dx\, dy\right)$$

If the profile is symmetric, then

$$Y = 8qi_{11}\alpha, \qquad X = 2q\left[c_0S + 2(i_{13} + 3\alpha^2 i_{11})\right]$$

Variation in the lift-drag ratio as a function of the angle of attack, may be represented in the form (Fig. 5, curve 1) /45

$$k = \frac{K}{K_m} = \frac{2\alpha_0}{1 + \alpha_0^2}, \qquad \alpha_0 = \frac{\alpha}{\alpha_m}$$

Here $K_m = 1/3\alpha_m$ is the maximum value of the lift-drag ratio and α_m is the corresponding angle of attack. In the case of a wedge-like profile of relative thickness 2c, we have $i_{11} = cS$, $i_{13} = c^3S$. Therefore

$$Y = 8qSc\alpha, \quad X = 2qS\left[c_0 + 2c(c^2 + 3\alpha^2)\right], \; \alpha_m = c\sqrt{1/3(1 + c_0/2c^3)} \tag{2.1}$$

When $c_0/c^3 < 4$, the distribution of volume is optimal on both sides of the velocity vector; when $c_0/c^3 = 4$ the upper side of the wing is directed along the flow, i.e., the entire volume is below the velocity vector[1]. If the coefficient of detrimental drag is known, the optimal relative thickness and the corresponding lift-drag ratio are $c_m = (c_0/4)^{1/3}$, $K_{mm} = 2^{2/3}/3c_0^{1/3}$,

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[1] In [7] it is assumed a priori that the upper part is plane and directed along the flow.

In this case the generatrixes of the upper side are directed along the flow.

$$\frac{v^2}{c_y} = \frac{f_1^2(\omega)}{f_2(\omega)}$$

...fect of relative thickness on the maximum lift-drag ratio is equal to ...5, curve 2)

$$k_m = \frac{K_m}{K_{mm}} = \left(\frac{3c^\circ}{2+c^{\circ 3}}\right)^{1/2}, \qquad c^\circ = \frac{c}{c_m}.$$

If the thickness is equal to the optimum thickness, the relative friction drag is 1/3 of the total drag [1,4]. Variation in the lift-drag ratio as a function of characteristic parameter for a wedge-type profile is

$$\frac{K_m v}{l_0'^2} = \begin{cases} (27+g_2)^{-1/2} & (r=1) \\ (12+{}^9/_8 g_2)^{-1/2} & (r={}^1/_2) \end{cases} \tag{2.2}$$

The corresponding curves are shown by the dot-dash line in Figure 4. The difference between the lift-drag ratios of wings with the optimal and wedge-type profiles is slight[1]. Thus the lift-drag ratio of a wing with cylindrical surfaces is quite close to that of a wing of optimum profile of longitudinal cross-section. We see in [4] that in the case of a flat "homothetic" body a conic surface is optimal.

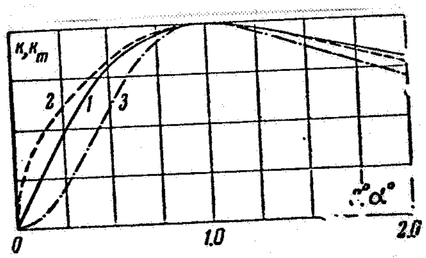

Figure 5

If the wing volume is not known, the optimal surface for the lower side of the wing is a cylindrical surface with a generatix of constant slope α. Consequently,

$$Y = 2qS\alpha^2, \quad X = 2qS(c_0+\alpha^3), \quad K = \alpha^2/(c_0+\alpha^3)$$

$$K_m = {}^2/_3\alpha_m = \frac{2^{2/3}}{3c^{1/3}}, \quad \alpha_m = (2c_0)^{1/3}, \quad k = \frac{K}{K_m} = \frac{3\alpha^{\circ 2}}{1+2\alpha^{\circ 3}}$$

The relation k = k (α°) is shown in Figure 5 (curve 3).

When the volume is not known the distribution of wing area over the span has no effect on the lift-drag ratio. On the other hand when the volume is known ($c_0/c^3 < 4$) the distribution of area is governed by the relative thickness $c = v/s_2$, and rather strongly so, not only when l_0' is known, but also when the wing area is known. Thus, for example, when the parameter $g_2 = 0$ the lift-drag ratio of the wing whose plane form is characterized by the exponent /46 r = 1/2 is 17% less than that of a delta wing (r = 1).

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[1] It is obvious from equations (2.2) that the parameter $V2/3/S_p$, where S_p is the area of the surface over which flow takes place, cannot be a characteristic parameter in the general case because it does not contain c_0. Also power which contains the relative thickness is different for bodies of different shape.

Since in the case of optimal profile of the longitudinal cross-section, the variable functionals (1.10; 1.14) contain only the value of the wing chord $l(y)$; the variational problem with respect to $l(y)$ has no solution in general or has the trivial solution $l(y) = l_0$. Thus, in order to determine it we must have additional information, which will be presented in the following sections.

It is obvious from the above that in selecting the optimal dimensions of the wing it is quite permissible to consider a wing which is made up of cylindrical surfaces, i.e., to make use of equations (2.1). If the lift and the wing volume are known, the drag is given by the expression

$$\frac{X}{2qb^2} = \left[c_0 + 2\left(\frac{v}{s_2}\right)^3 \right] s_1 + \frac{3}{8}\,\frac{c_y{}^2}{v}\,\frac{s_2}{s_1}, \tag{2.3}$$

Variation in the quantity which is proportional to the wave drag

$$\Delta = (X/2qb^2 - c_0 S/b^2)\, l_0{}'^5 v^{-3}$$

as a function of the parameter g_1 is shown in Figure 6 by the dot-dash line together with the corresponding curves for a wing with optimal profile. The difference in the lift-drag ratios here is somewhat greater than in Figure 5. It follows from equation (2.3) that when Y and V are known it is possible to have optimal semi-span b and area distribution over the span.

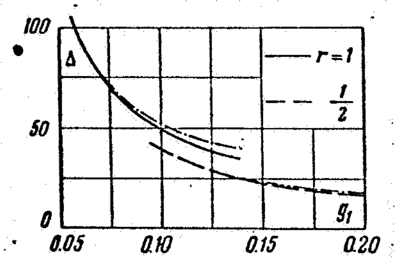


Figure 6.

If the problem concerning the optimum length of the wing is meaningful its solution requires that we take into account the variation in the friction coefficient as a function of length.

3. <u>The shape of the leading edge of the wing.</u> For small angles of attack the thermal flux attains its greatest value at the leading edge. Consequently its shape must be selected in such a way that the temperature does not exceed the permissible value, or, if this is impossible, the given temperature must be maintained by a cooling system. In the first case the capabilities of a material will be completely utilized to reduce the wave drag of the edge in the case when the temperature along the entire length of the edge has a maximum permissible value based on strength conditions. We shall limit ourselves to this case and will consider the drag of the edge independently of the remaining part of the wing. This is permissible in the case where the volume is not known or where the given volume can be found on the basis of the bottom region. Let us assume that the leading edge has the shape of a semicircle in the cross section normal to the arc $\sigma(\phi)$ (Fig. 2). Then its surface can be given in a parametric form

$$x = x_0(\varphi) - r(\varphi)\cos\theta\cos\varphi, \quad y = y_0(\varphi) + r(\varphi)\cos\theta\sin\varphi, \quad z = r(\varphi)\sin\theta \tag{3.1}$$

The cosine of the angle between the velocity vector and the line normal to the surface is

$$\cos(n, V) = \frac{[r' \sin\varphi + \cos\theta\cos\varphi(\sigma' + r\cos\theta)]\cos\alpha - \sin\theta(\sigma' + r\cos\theta)\sin\alpha}{\sqrt{r'^2 + (\sigma' + r\cos\theta)^2}}$$

When the angle of attack is $\alpha = 0$, drag of the leading edge is

$$X_0 = 2q \int_0^{\varphi_1} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{r[r' \sin\varphi + \cos\theta\cos\varphi(\sigma' + r\cos\theta)]^3}{r'^2 + (\sigma' + r\cos\theta)^2} \, d\varphi \, d\theta \tag{3.2}$$

The condition that the maximum temperature remains constant at the edge can be approximated in the form

$$\cos\varphi \left(\frac{1}{2}\left(\frac{1}{r} + \frac{1}{\sigma'}\right)\right)^{\frac{1}{2}} = \frac{1}{\sqrt{R}}, \qquad r = \frac{R\sigma' \cos^2\varphi}{2\sigma' - R\cos^2\varphi} \tag{3.3}$$

Here R is the average radius of curvature of the surface permitted by /47
the strength of the material. It is obvious that the solution of the variational problem based on equation (3.2) is complicated in the general case by the difficulties of integrating over θ.

Let us assume that $r' \ll \sigma'$ and that we can disregard this quantity. Then

$$X_0 = \frac{8qR}{3} \int_0^{\varphi_1} \frac{\sigma'^2 \cos^5\varphi}{2\sigma' - R\cos^2\varphi} \left(1 + \frac{9\pi}{32} \frac{R\cos^2\varphi}{2\sigma' - R\cos^2\varphi}\right) d\varphi$$

It can be shown that the second derivative of the expression under the integral sign with respect to σ' will be greater than zero. Consequently the Legendre condition is satisfied.

The external value may include the region of straight line L if in this case the point ϕ_1 is not an angular point. Then the total drag of the edge will be

$$X_0 = \frac{8qR}{3} \left\{ \int_0^{\varphi_1} \frac{\sigma'^2 \cos^5\varphi}{2\sigma' - R\cos^2\varphi} \left(1 + \frac{9\pi}{32} \frac{R\cos^2\varphi}{2\sigma' - R\cos^2\varphi}\right) d\varphi + \frac{L\cos^5\varphi_1}{2} \right\} \tag{3.4}$$

The width and length of the wing are given by

$$b = \int_0^{\varphi_1} \sigma' \cos\varphi \, d\varphi + L\cos\varphi_1, \qquad l_0 = \int_0^{\varphi_1} \sigma' \sin\varphi \, d\varphi + L\sin\varphi_1$$

It is possible to formulate various isoperimetric variational problems. Euler's equations for these problems differ in the free term P (ϕ, ϕ_1)

$$\frac{\sigma'(\sigma' - R\cos^2\varphi)}{(2\sigma' - R\cos^2\varphi)^2} - \frac{9\pi}{32}\frac{R^2\sigma'\cos^4\varphi}{(2\sigma' - R\cos^2\varphi)^3} - P = 0 \tag{3.5}$$

If there are no relationships and the boundary condition at the point ϕ_1 is natural, Euler's equation has the solution

$$\sigma' = {}^3\!/_4 R\cos^2\varphi\left(1 + \sqrt{{}^1\!/_9 + {}^1\!/_4\pi}\right) = 1.46\,R\cos^2\varphi$$

Equation (3.5) is cubic in the general case and its solution depends on the sign of the discriminant

$$D = -\frac{27R^6\cos^{12}\varphi}{16(1-4P)^3}\left[\left(\frac{9\pi}{32}\right)^2(1-4P) - \left(\frac{1}{3} + \frac{3\pi}{16}\right)^3\right]$$

If D < 0 there is one real root:

$$\sigma' = \frac{R\cos^2\varphi}{2}\left\{1 + \frac{1}{2}\left(\frac{{}^9\!/_4\pi}{1-4P}\right)^{1/3}\left[\left(1 + \left(1 - \frac{(\pi + {}^{16}\!/_9)^3}{12\pi^2(1-4P)}\right)^{1/2}\right)^{1/3} + \right.\right.$$
$$\left.\left. + \left(1 - \left(1 - \frac{(\pi + {}^{16}\!/_9)^3}{12\pi^2(1-4P)}\right)^{1/2}\right)^{1/3}\right]\right\}$$

If D > 0 then all three roots are real:

$$\sigma' = \frac{R\cos^2\varphi}{2}\left(1 + \left(\frac{{}^4\!/_3 + {}^3\!/_4\pi}{1-4P}\right)^{1/2}\cos\psi\right), \qquad \cos 3\psi = \frac{9\pi\sqrt{1-4P}}{4({}^4\!/_3 + {}^3\!/_4\pi)^{3/2}}$$

If we take into account the continuity condition for the curvature at the point ϕ_1 we have for the function P:

The external value does not contain a rectilinear section; width and length of the wing are known (λ, μ are indeterminant factors)

$$P = -2(\lambda\cos\varphi + \mu\sin\varphi)\,/\cos^5\varphi$$

The extremal value contains a rectilinear region; either the length or width is given. Then

$$P = \frac{\cos^4\varphi_1}{4\cos^4\varphi}, \qquad P = \frac{\sin\varphi\cos^5\varphi_1}{4\sin\varphi_1\cos^5\varphi}$$

The analysis of the results obtained shows that in the general case the results contradict the hypothesis $r' \ll \sigma'$ except in the vicinity of the stagnation point. Thus we can only conclude that at the stagnation point the optimum radius of curvature is $\sigma_0' \approx (1 - 2.5)R$. It decreases further and only near the point ϕ_1 does it increase to infinity if the extremal value becomes a straight line. The width of the curvilinear section of the edge appears to be of order R. To solve the problem concerning the optimum leading edge it is necessary to use the method of direct variation. /48

As an example let us consider the schematized leading edge consisting of a torus with radius R_0 and $r_0 = RR_0/(2R_0 - R)$ and a rectilinear region with a radius $r_1 = 1/2R \cos^2\phi_1$. The width, length and area of the wing are

$$b_0 = R_0 \sin\varphi_1 + L\cos\varphi_1,\quad l_0 = R_0(1 - \cos\varphi_1) + L\sin\varphi_1$$
$$S = {}^1/_2 R_0^2(\varphi_1 - \sin\varphi_1\cos\varphi_1) + L(R_0\sin\varphi_1 + {}^1/_2 L\cos\varphi_1)\sin\varphi_1$$

For the drag of the leading edge we obtain

$$X_0 = \frac{8qR}{3}\left[\frac{R_0^2}{2R_0 - R}\left(1 + \left(\frac{9\pi}{32} - 1\right)\frac{R}{2R_0 - R}\right)\left(\sin\varphi_1 - \frac{\sin^3\varphi_1}{3}\right) + \frac{L\cos^5\varphi_1}{2}\right]$$

If the area and length of the wing are known, then

$$R_0 = \frac{l_0}{2}\left\{\frac{\operatorname{tg}{}^1/_2\varphi_1}{\operatorname{tg}{}^1/_2\varphi_1 - {}^1/_2\varphi_1} - \left(\left(\frac{\operatorname{tg}{}^1/_2\varphi_1}{\operatorname{tg}{}^1/_2\varphi_1 - {}^1/_2\varphi_1}\right)^2 + \frac{2\operatorname{ctg}\varphi_1 - 4S/l_0^2}{\operatorname{tg}{}^1/_2\varphi_1 - {}^1/_2\varphi_1}\right)^{1/2}\right\}$$

If, on the other hand, the area and the width are known, then

$$\frac{R_0}{b} = \left(\frac{\operatorname{tg}\varphi_1 - 2S/b^2}{\operatorname{tg}\varphi_1 - \varphi_1}\right)^{1/2}$$

When the width is $b/R \approx 1.2$ the optimum edge is rectilinear ($\phi_1 = 0$) but when b/R are great, minimum drag is achieved when $\varphi_1 \approx 80°$, $R_0/R \approx 1$. The quantity $R_0/R \approx 1$ is also optimal when the area and the width are given or when the area and the length of the wing are known. The optimum angle ϕ_1 in this case varies. The optimum ratio R_0/R increases when the given l and $2S/l^2$ increase.

If the leading edge of the wing is thin and if we can assume that $r' \ll \sigma'$ and $r' \ll \sigma'$ the drag for angle of attack α is given by [5].

$$X_0 = 2qR \int_0^{\varphi_1} [{}^2/_3(1 - \cos^2\alpha \sin^2\varphi)^{3/2} + \cos\alpha \cos\varphi (1 - \cos^2\alpha \sin^2\varphi) - {}^1/_3 \cos^3\alpha \cos^3\varphi] \frac{\sigma'^2 \cos^2\varphi \, d\varphi}{2\sigma' - R\cos^2\varphi}$$

It is obvious from (3.4) and (3.6) that the extremal problem of the minimum temperature (the magnitude of 1/R) when the drag is known, is equivalent to the problem of the minimum drag at a given temperature.

4. <u>The shape of a wing width of known volume</u>. It is obvious that in the case where the wing volume is known and the bottom region cannot be used to determine this volume (the trailing edge is straight) it is necessary to seek the minimum total drag of the leading edge and of the remainder of the wing in order to determine the wing shape. This problem has to be solved by the method of direct variation. Let us assume that the lower and upper surfaces of the wing are cylindrical and that the slopes of the generatrixes with respect to the velocity vector are α and β, respectively. If we designate $\xi = 1/2\ (\alpha + \beta)$ and $\eta = 1/2(\alpha - \beta)$ assuming that edge is thin, we have the following equations for total drag, lift and wing volume. /49

$$c_x = \frac{X}{2qb^2} = [2\xi(\xi^2 + 3\eta^2) + c_F]\, s_1 + \frac{4J_1R}{3b}$$

$$c_y = \frac{Y}{2qb_2} = 4\xi\eta s_1, \qquad v = \frac{V}{b^3} = \frac{2l_0'J_2R}{b} + \xi s_2 \tag{4.1}$$

$$J_1 = \int_0^1 \frac{r}{R} \cos^2\varphi \, dy', \qquad J_2 = \int_0^1 \frac{r}{R} l' dy' \tag{4.2}$$

The maximum lift-drag ratio $K_m = 1/3\eta_m$ will exist when the angle of attack is

$$\eta_m = \left(\frac{\xi^2}{3} + \frac{c_F}{6\xi} + \frac{2J_1R}{9\xi s_1 b}\right)^{1/2} \tag{4.3}$$

The volume should be considered as known if

$$v \geqslant s_2\left(\frac{1}{4} c_F + \frac{J_1R}{3s_1b}\right)^{1/3} + \frac{2l_0'J_2R}{b} \tag{4.4}$$

Let us consider two families of wings of known plan form:

$$x_0 = l_0 y'^n, \quad s_1 = \frac{n l_0'}{n+1}, \quad s_2 = \frac{2n^2 l_0'^2}{(n+1)(2n+1)}, \quad n \geqslant 2$$

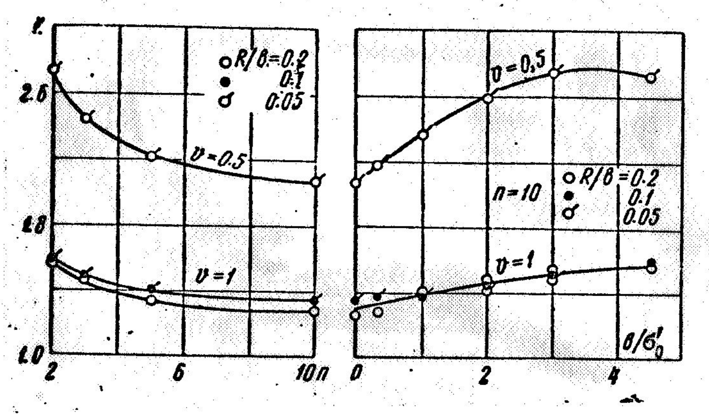
and the radius of curvature for the contour at the stagnation point is

$$\sigma_0' = b / 2l_0' \text{ при } n = 2, \qquad \sigma_0' = \infty \text{ при } n > 2$$

$$x_0 = l_0' y'^n + \frac{b(y'^2 - y'^n)}{2\sigma_0'/b}, \quad s_1 = \left[\frac{n}{n+1} - \frac{n-2}{6\sigma_0' l_0'(n+1)/b}\right] l_0'$$

$$s_2 = \left[\frac{2n^2}{(n+1)(2n+1)} - \frac{2n(n^2-4)}{3(2n+1)(n+1)(n+3)} \frac{b}{l_0'\sigma_0'} + \frac{2(n-2)^2 b^2}{5(2n+1)(n+3)(2\sigma_0' l_0')^2}\right] l_0'^2$$

We assume in both cases that the width of the wing and its area are known. The results of calculations for $s_1 = 1.5$, $c_F = 0.001$ are shown in Figure 7. Under the specified conditions a decrease in l_0' caused by an increase in n in case (1) and by an increase in σ_0' in case (2) results in a reduction in the lift-drag ratio. For large relative volumes v, a decrease in the permissible temperature of the leading edge (an increase in R) does not lead to a large decrease in the lift-drag ratio.



The evaluation of the maximum possible value of the lift-drag ratio using Newton's Law, obtained in this work is very close to that found for pointed bodies constructed with the aid of the stream surfaces of two-dimensional flows [6]. This makes it possible to verify the computations based on Newton's Law, especially since the optimum shapes are close to pyramidal.

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